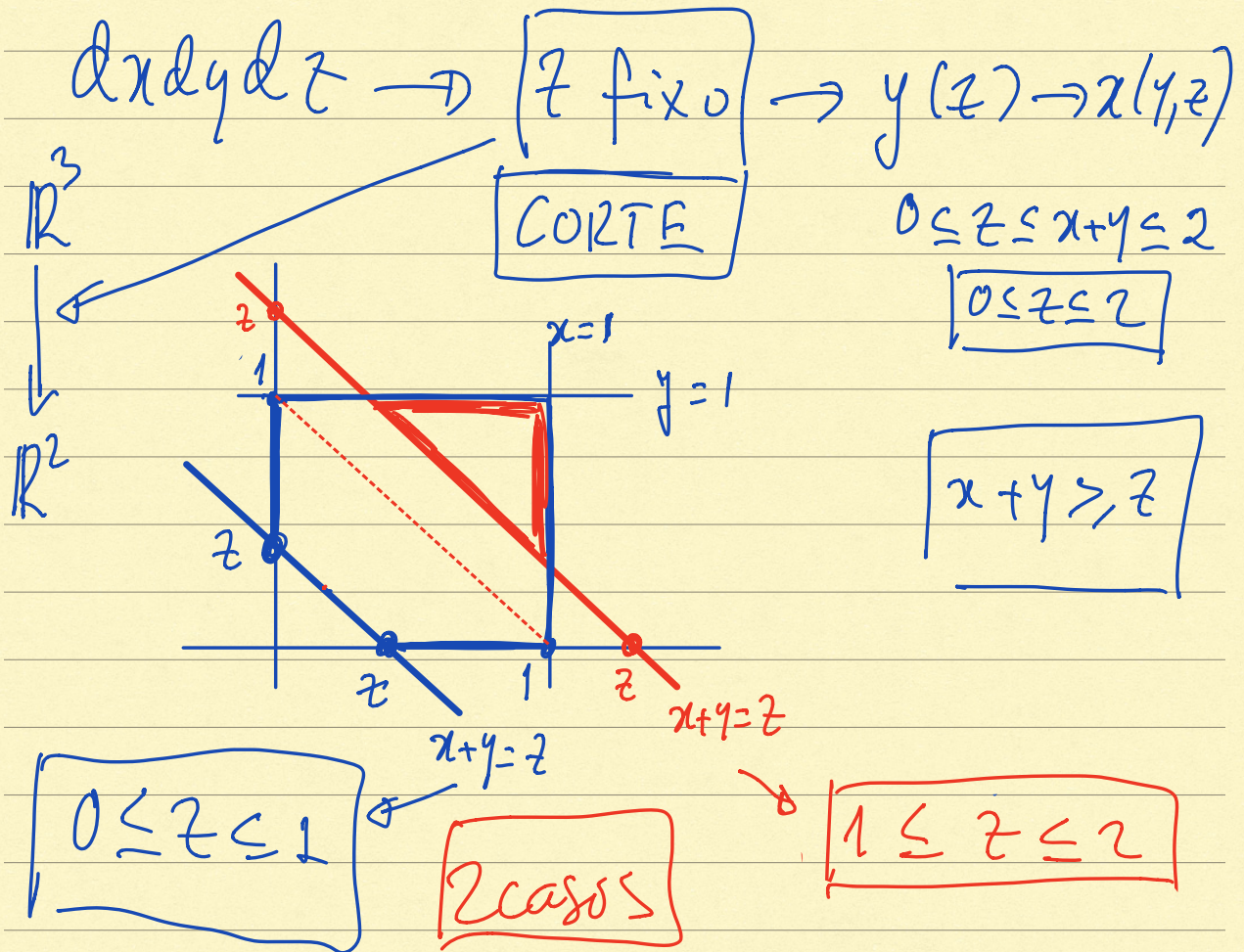


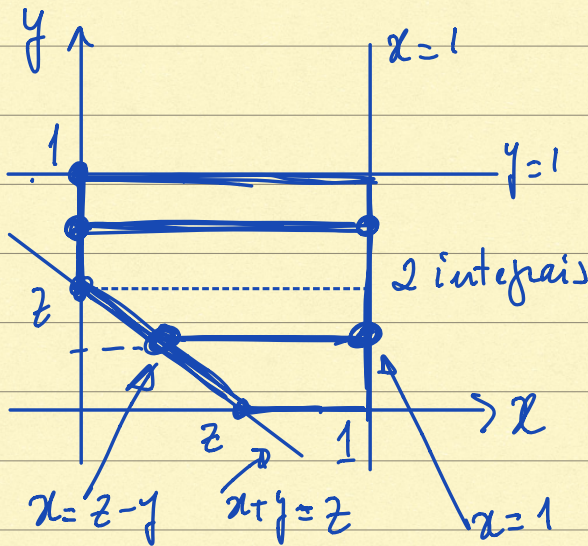
# Integrais Múltiplos (iterados) : $\mathbb{R}^3$

Exemplo:

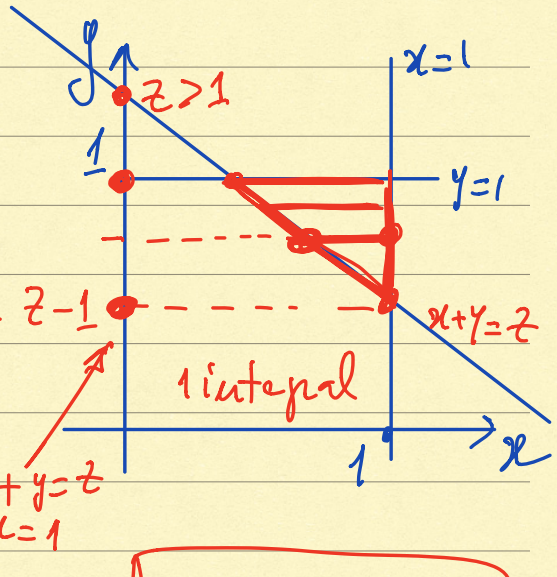
$$X = \left\{ (x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 1; 0 \leq y \leq 1; 0 \leq z \leq x+y \right\}.$$







$$\boxed{0 \leq z \leq 1}$$



$$\boxed{1 \leq z \leq 2}$$

$$0 \leq x \leq 1; \quad 0 \leq y \leq 1; \quad 0 \leq z \leq x+y$$

$$\int \left( \int \left( \int dx \right) dy \right) dz$$

X

$$z \text{ fixo} \rightarrow y(z) \rightarrow x(y, z)$$

$$\int_0^1 \left( \int_0^z \left( \int_{z-y}^1 dx \right) dy \right) dz + \int_0^1 \left( \int_z^1 \left( \int_0^1 dx \right) dy \right) dz$$

+



$$+ \int_1^2 \left( \int_{z-1}^1 \left( \int_{z-y}^1 dx \right) dy \right) dz$$

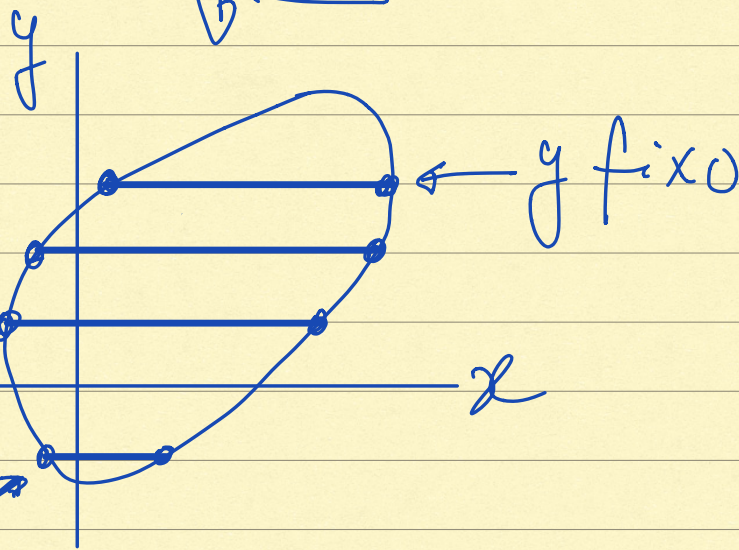
Resumo: Integral triplo iterado

$dx dy dz$ :  $z$  fixo  $\rightarrow$   $y(z)$   $\rightarrow$   $x(y, z)$

~~$\mathbb{R}^3$~~

CORTE em  $X$

CORTE ( $\mathbb{R}^2$ )



$x$  em  $\mathbb{R}$



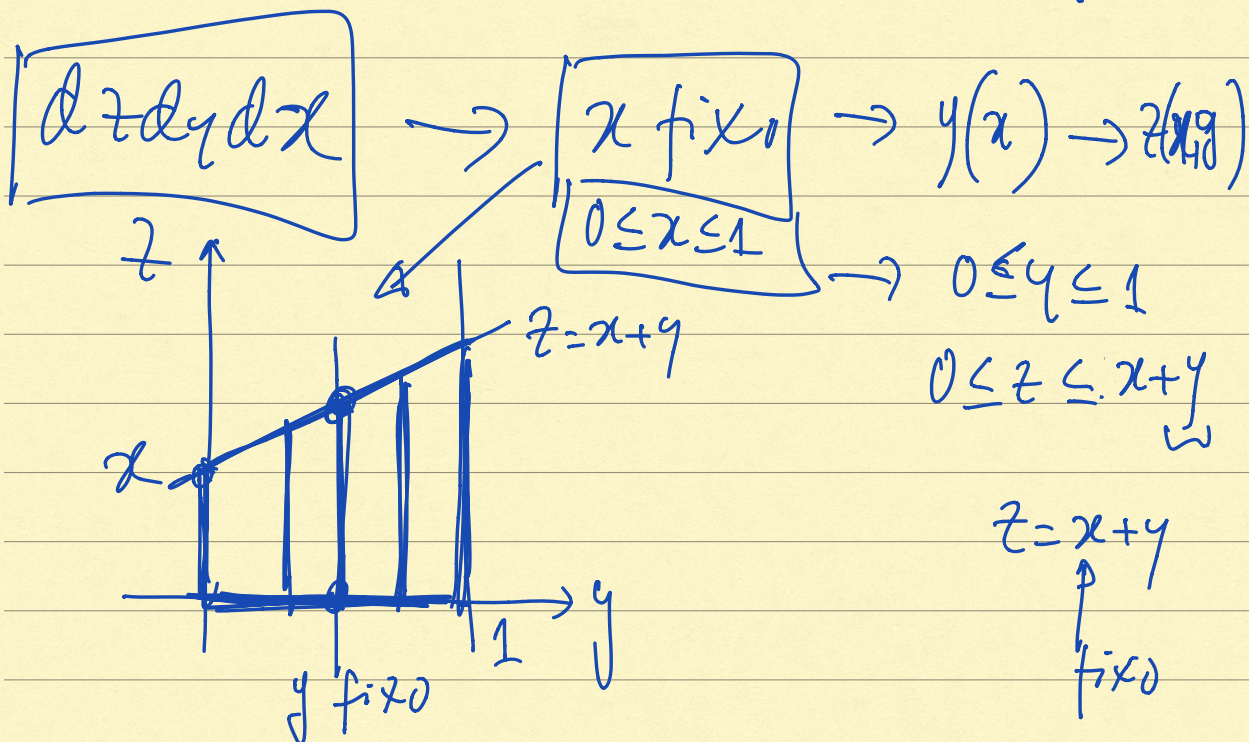
$$X \iint\int f(x, y, z) dx dy dz$$

$$f(x, y, z) = x$$

Soma de 3 integrais triplas.

$$X: \boxed{0 \leq x \leq 1; 0 \leq y \leq 1; 0 \leq z \leq x+y}$$

Order simples:  $\boxed{dz dy dx}$   $x$  fixo  
 ou  $dz dx dy$   $y$  fixo





$$\int_X f = \iiint_X f = \int_0^1 \left( \int_0^1 \left( \int_0^{x+y} x dz \right) dy \right) dx \quad \mathbb{R}^3$$

$$= \int_0^1 \left( \int_0^1 (x(x+y)) dy \right) dx$$

$$= \int_0^1 \left( \int_0^1 (x^2 + xy) dy \right) dx \quad \mathbb{R}^2$$

$$= \int_0^1 \left( x^2 + \frac{x}{2} \right) dx \quad \mathbb{R}$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$



Aplicações:  $\int_X f \equiv$  integral de  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  em  $X \subset \mathbb{R}^n$ .

1) Massa:  $f \equiv \sigma: \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $\sigma \geq 0$   
densidade de massa.  $\swarrow$  "sigma"

$$\int_X f = \int_X \sigma$$

$$M = \int_X \sigma \quad \text{importante: } n=3$$

2) Centro de massa (em  $\mathbb{R}^3$ )

Ponto de coordenadas:  $(\bar{x}, \bar{y}, \bar{z})$

definidas por:

$$\bar{x} = \frac{1}{M} \int_X x \sigma$$

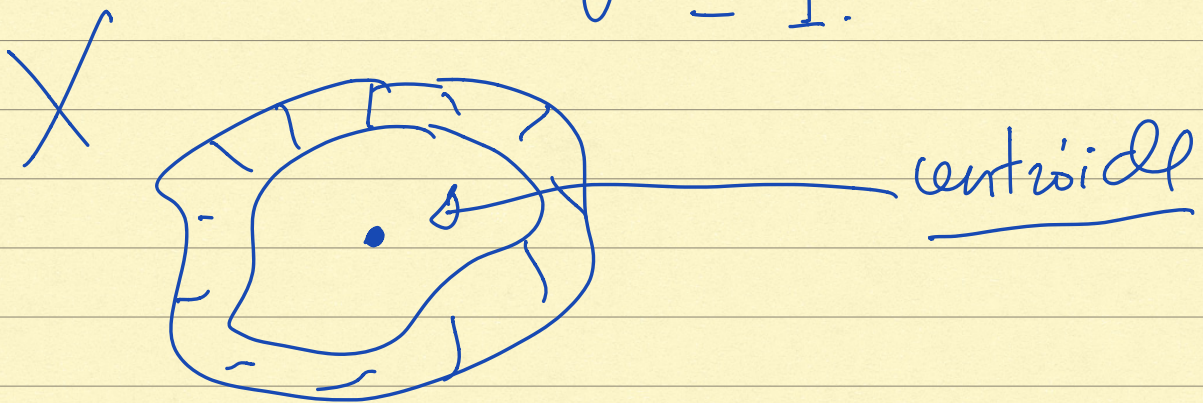


$$\bar{y} = \frac{1}{M} \int_X y \sigma$$

$$\bar{z} = \frac{1}{M} \int_X z \sigma ; \quad M = \int_X \sigma$$

Centróide  $\equiv$  Centro de massa

$$\sigma = 1.$$



3) Momento de inércia de X  
relativo a uma linha  
recta  $L \dots \rightarrow$  próxima a aula.